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Conjugated (Complementary) Coupling for Two Penrose Tiles Pair and their Elementary Unit Cell for Translation Tiles

Kung Chung Yuan*

Department of Electrical Engineering, National Chung Hsing University. 145, Xingda Road., South Dist. Taichung City 40227. Taiwan

***Corresponding Author:** Kung Chung Yuan, Department of Electrical Engineering, National Chung Hsing University. 145, Xingda Road., South Dist. Taichung City 40227. Taiwan, Tel: 886-4-22850359, E-mail: cykung@dragon.nchu.edu.tw

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Abstract

The Penrose tiling, which is one of the most famous and complex tessellations, has been drawn and represented in many different ways, with most in artistic form. Consequently, two different types of pentagonal Penrose tiles are constructed in this study in order to create translational periodic Penrose tiles (crystals). It is found that these two Pentagonal-shaped Penrose tiles can be coupled successfully without any defect and at different depths. Therefore, a model of Penrose multiple concentric nanotube is established to facility a further 3-DPenrose-tiling studies.

Keywords: Decagon; Penrose Tiling; Pentagon Shap; Translational Periodic Penrose Tiles; Unit Cell

Introduction

The Penrose tiling is one of the most well-known and complex tessellations. It has been drawn and depicted in numerous ways, with majority being in artistic form. Roger Penrose discovered a group of six tiles that force aperiodicity in 1973. He was eventually able to decrease the set to two tiles in 1974. A pair of Penrose tiles can have a variety of shapes, but the kite and the dart are the most typical. Roger Penrose discovered a group of six tiles that force aperiodicity in 1973. He was eventually able to decrease the set to two tiles in 1974. A pair of Penrose tiles can have a variety of shapes, but the kite and the dart are the most typical.

The five-fold symmetric Penrose tiling diagram has been the subject of various investigations in this area since it helps to explain the five-fold character of quasicrystals [1-3].

The present study strictly focuses on defined error-free pentose tiling and how to extend tiling to infinity in a systematic and orderly manner. One of the systematic ways to draw defect-free Penrose tiles is to ensure that the tile is defect-free under strict definitions with a perceivable and repeatable internal structure that can be extended to infinity.

Penrose Tiling Constructed of Two Rhombuses

Six different decagons can be formed from two rhombuses with acute angles of 36 and 72 degrees (Figure 1). Using these two rhombuses, [4] created the well-known Penrose-tile in 1974, which can be extended indefinitely, as shown in Figure 1. Following the original pattern configuration found by Penrose, several decagonal shapes were added outwards, resulting in a regular pentagonal pattern with five decagons on each side, as shown in Figure 2.The pattern consists of three decagons of types a, b, and c (or might be only type-a and type-b two decagons). The pentagon shape in Figure 2a appears a bit bloated. At the same time, a counterpart to a bulky Penrose tile was found, with a slightly slimmer pentagonal shape, as shown in Figure 2b.

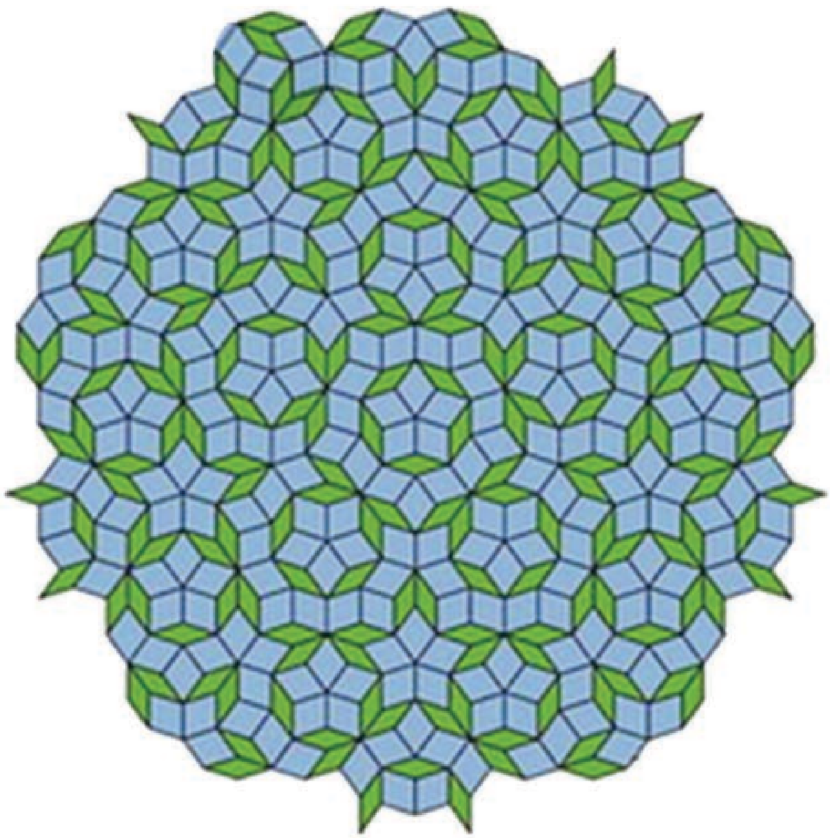

Figure 1: Original Penrose tile

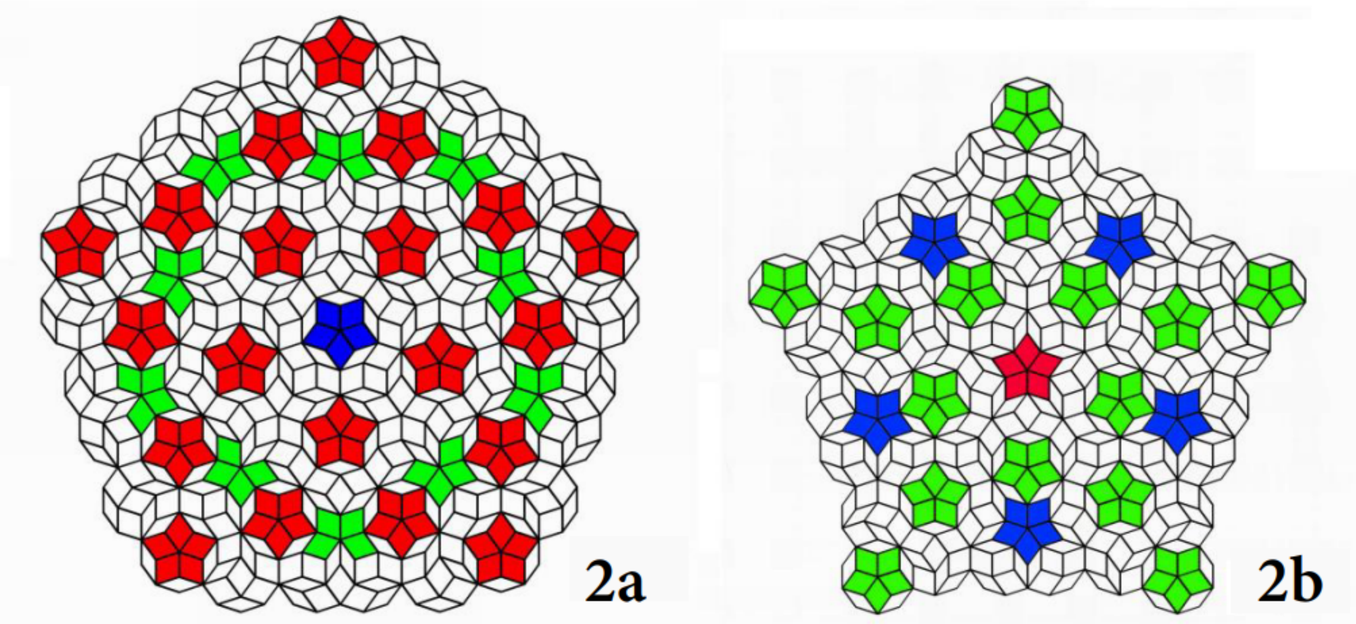

Figure 2: A regular pentagonal pattern with five decagonal on each side, (a) Bloated, (b) slim

All the vertices of each pentagonal Penrose tile are designated as a-type decagons (it is important to note that these five vertices can be substituted with other types of decagons, which may produce extensions in a variety of ways).To create the framework for future coupling alignment, type-a decagons in the dilated pentagonal tile (Figure 2a) are specially designated with red stars, while type-a decagons on the slim pentagonal tile (Figure 2b) are marked in green. The central regions of both bloated (dilated) and slim Pentagons are marked in dark blue for the purpose of coupling alignment.

Several examples of pair coupling are shown Figures 3, 4 and 5. Some of them still retain the mismatched regions to demonstrate the difficulty of producing defect-free Penrose tiles. The aforementioned various couplings may result in defects in the overlapping seam area. In most cases, these coupling defects can be easily removed to achieve even larger zero-defect Penrose tiles.

The examples of bottom-to-bottom opposite direction coupling in Figure 3, vertex-to-vertex opposing direction coupling in Figure 4, and bottom-vertex same direction coupling in Figure 5 illustrate in details these Bloated-slim types of coupling. Figure 3a shows a simple skin coupling; only the decades on outer layer are involved and perfectly coupled, necessitating no revision. Figure 3b shows a deeper coupling involving the outer three layers. Figure 3b1 shows the coupling before the revision, still retaining the remaining mismatched areas, while Figure 3b2 shows coupling after revision with the new type-e ingenerated (*created) to meet the defect-free requirements. Figure 3c shows an even deeper coupling covering almost all the region of the slim-type tile (Figure 3c1)and is able to produce an incredible defect free revision(Figure 3c2). Figure 3d shows a completed coupling before and after the revision of these two pairs, while Figure 3d2 shows a circular Penrose tile. Figure 3e and f shows a slanted shift coupling of these Penrose tile pairs which also ingenerated a type-e decagon to meet the defect free requirement. Figure 4a shows the result of a deep coupling before revision, while Figure 4b and c shows the same coupling after revision.

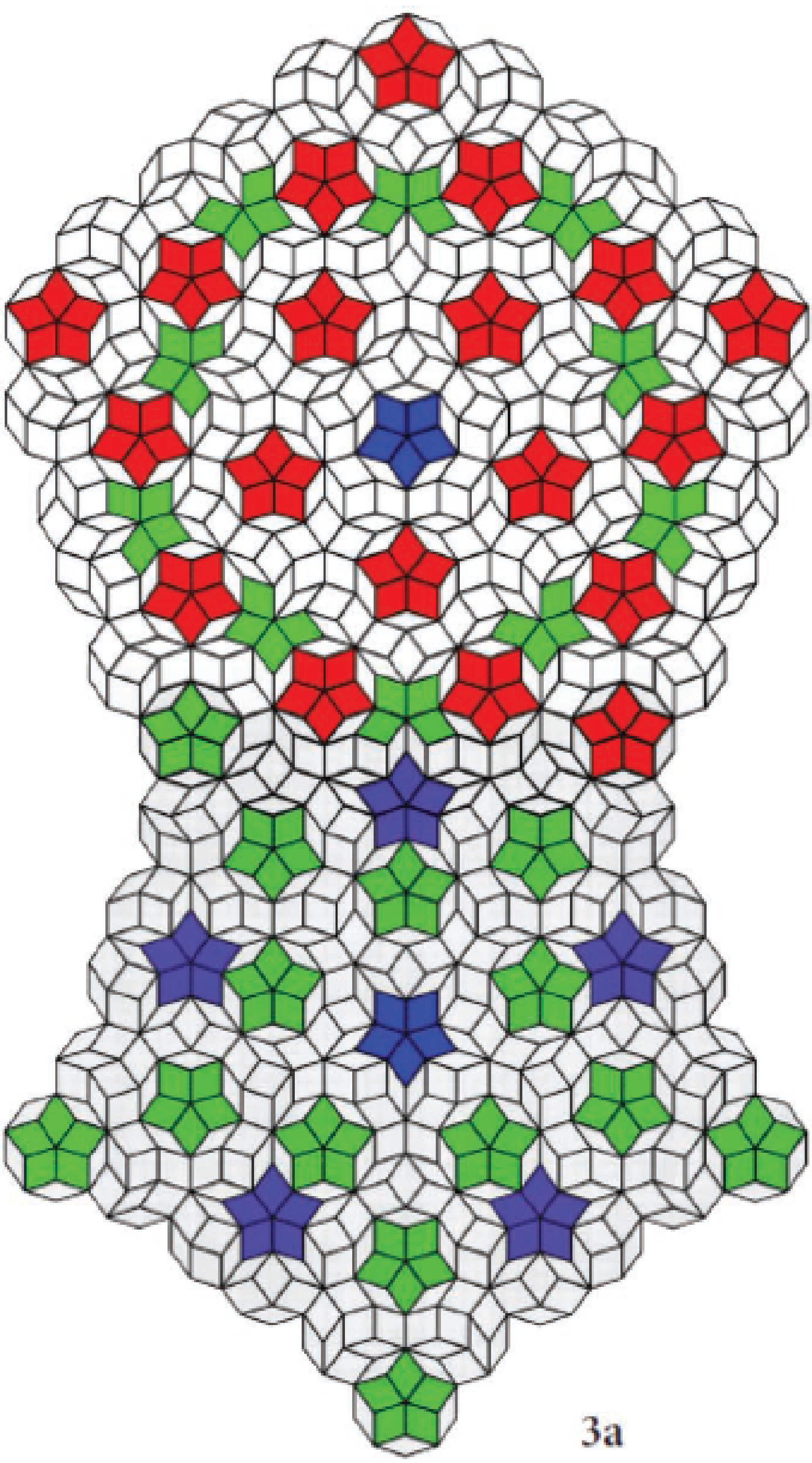

Figure 3: The examples of bottom-to-bottom opposite direction coupling type, (3a) Shows a simple skin perfectly coupling

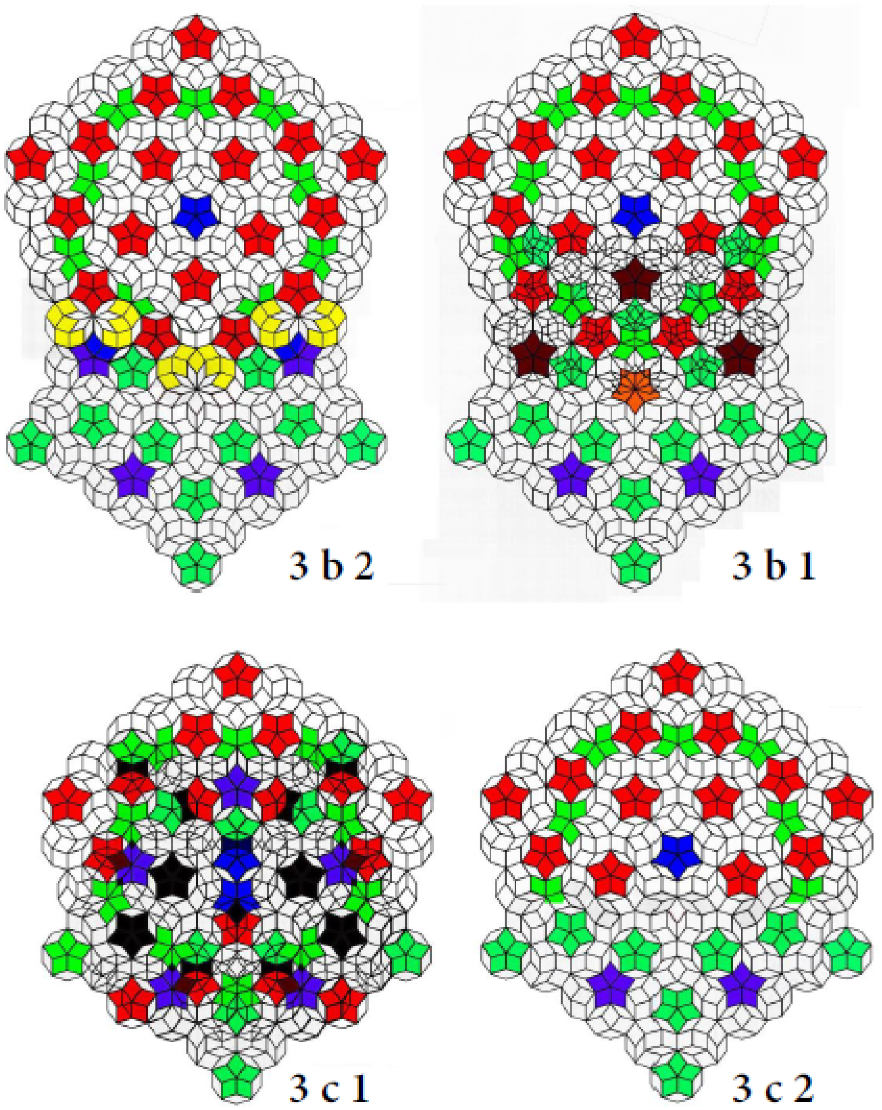

Figure 3b: Deeper coupling, with outer three layers involved, 3b1: coupling before revision remaining mismatched regions and figure (3b2) is after revision, (3c) shows an even deeper coupling, covers almost all the region slim type tile, (3c1) before revision, (3c2) after revision

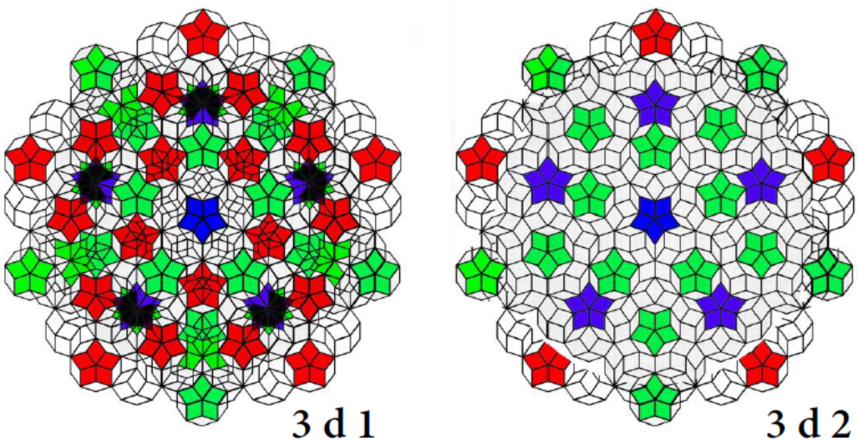

Figure 3d: Completed coupling (3d1) before and (3d2) after revision revealed a circular Penrose tile.

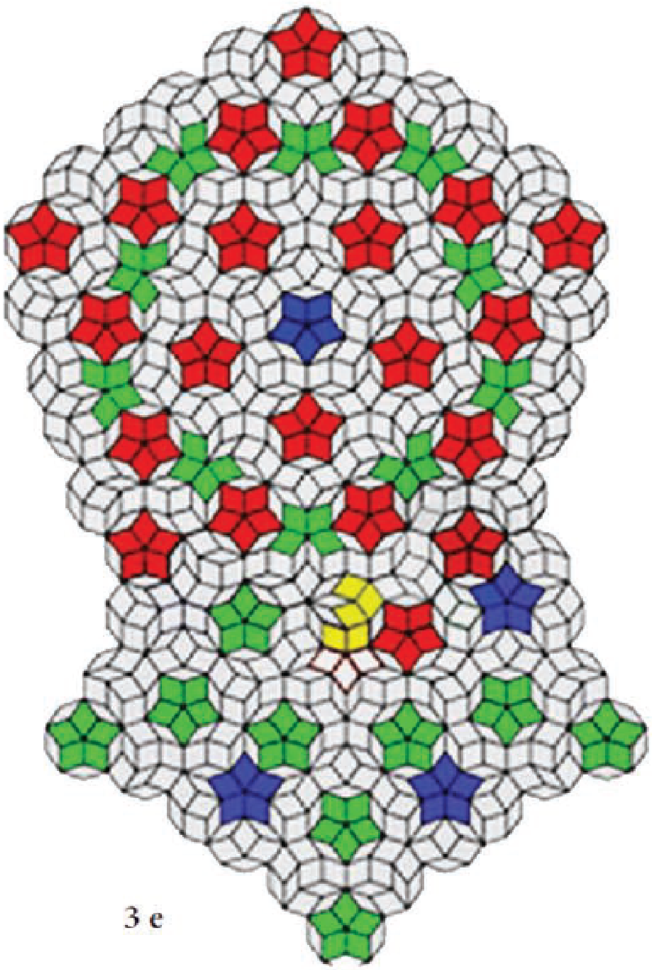

Figure 3e: A slanted shift coupling

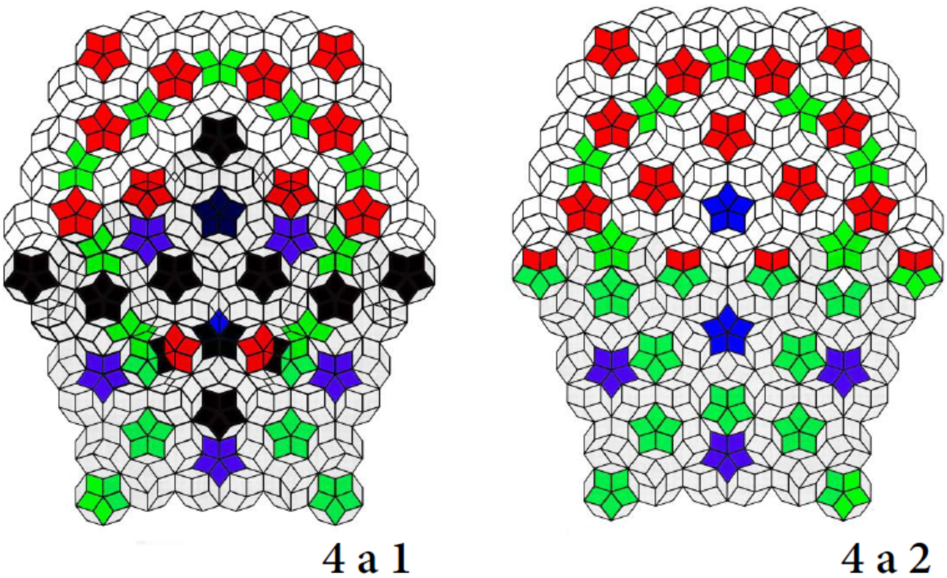

Figure 4: The examples vertex- to-vertex opposite direction coupling type Figure 4a1: Result of a deep coupling before revision, Figure 4a2: After revision.

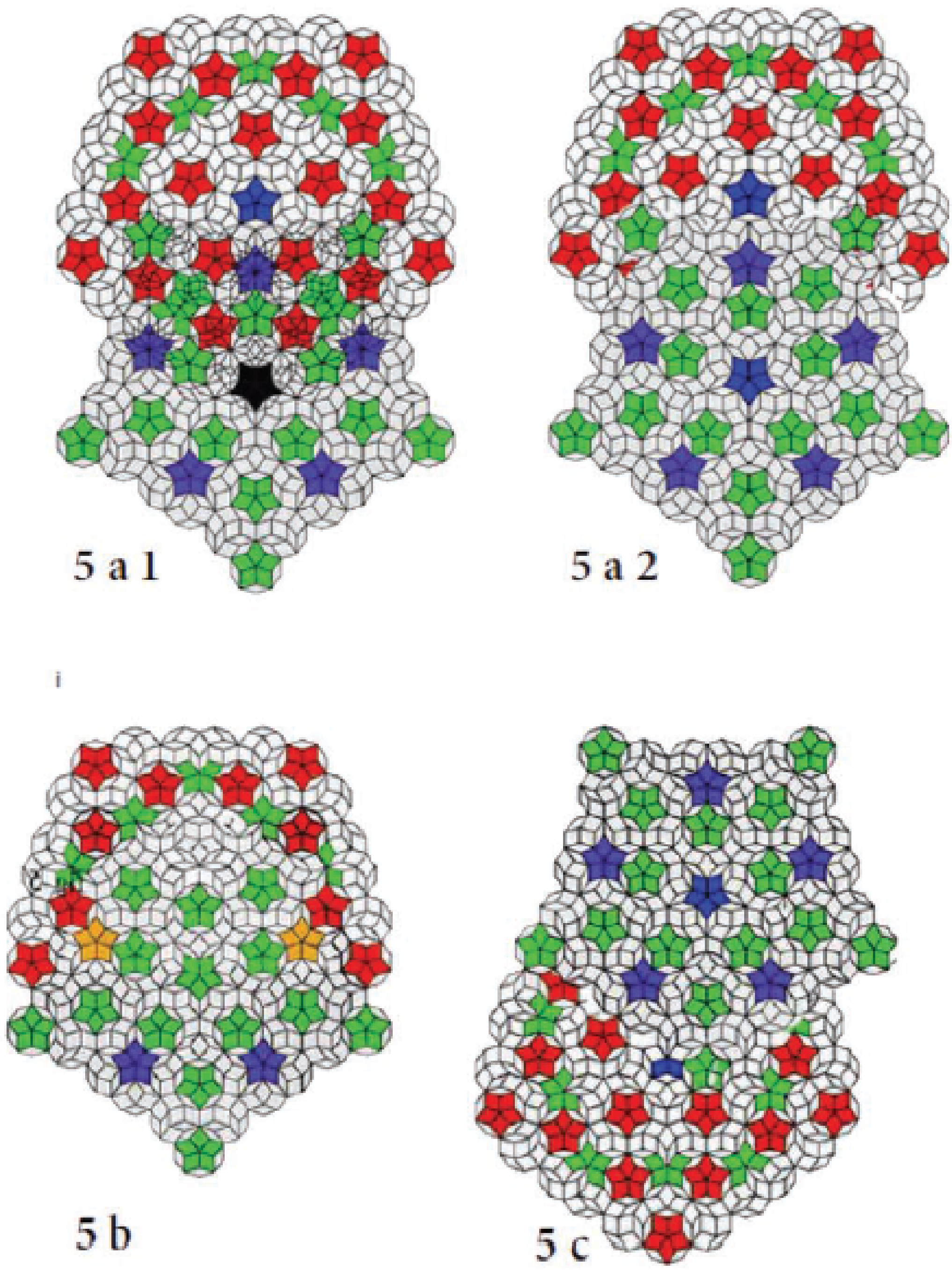

Figure 5: Bottom-vertex coupling in the same direction type. 5a1: Before revision 5a2: After revision, (5b) A deeper coupling after revision, (5c) Slant shift coupling after revision

Figure 5 shows the coupling results of the two Penrose tiles on the same direction. The Figure 5 demonstrates the bottom-vertex coupling type: 5(a1) before revision, 5(a2) after revision, (5b) a deeper coupling after revision, (5c) slant shift coupling after revision. The majority of defects in overlapped coupling can be fixed with some effort, as shown in Figures 3 and 4, but the process is not straightforward. The defects are left for readers to revise them and will not be discussed in this article. In fact, ignoring the defect ingenerated, all types of coupling can be performed easily for these two types of Penrose tiles, as shown in the case in Figure 5.These two Penrose tiles can be referred to as mutual conjugates since they have a high likelihood of coupling with one another under numerous distinct coupling conditions.

The different coupling mentioned above may result to defects in the overlapping seam area, and in most cases, these coupling defects can be easily removed to achieve a larger zero-defect Penrose tiles*(It should be noted that the traditional Penrose tiles allow the existence of defects).

All of these coupled pairs can themselves be transnationally (or slanted shift) coupled as shown in Figure 6a and b to form translational aperiodic Penrose tiles. In this study, Figure 3c2 is utilized to illustrate self-coupled to infinity (as shown in Figure 7a) and from that point, the elementary unit cell is extracted (Figure 7b). Thus, this elementary unit cell can be tessellated to construct a translational periodic Penrose tiling. In fact, more than one of unit cells can be extracted from figure 7a.

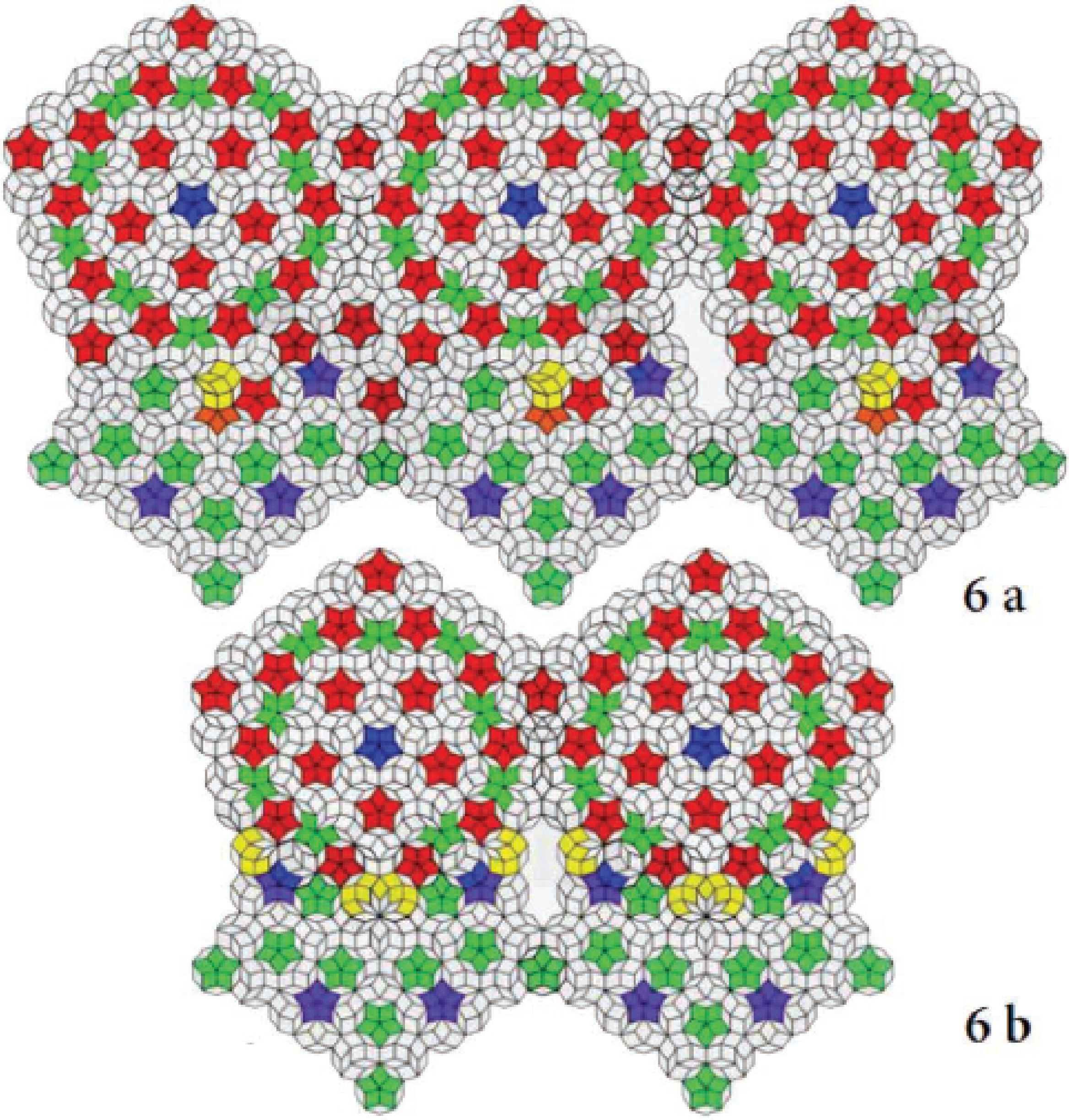

Figure 6: Coupled pairs can be translational (or slanted shift) coupled by their self as shown in figure 6 a,b

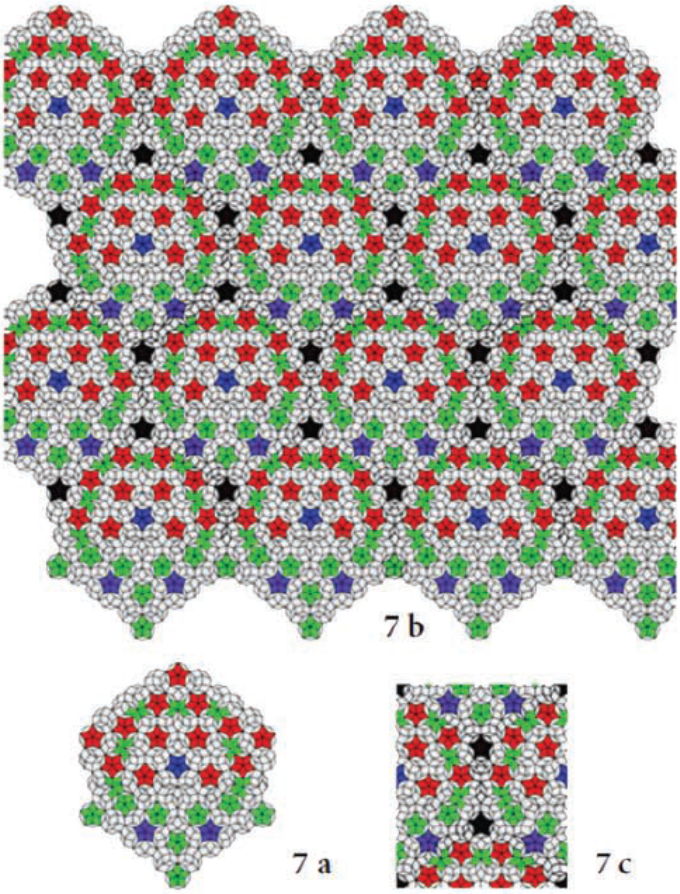

Figure 7: 7a: Tile in figure 3c2 self-coupled to infinity, Figure 7b: Anunit cell extracted from tile in figure 7c

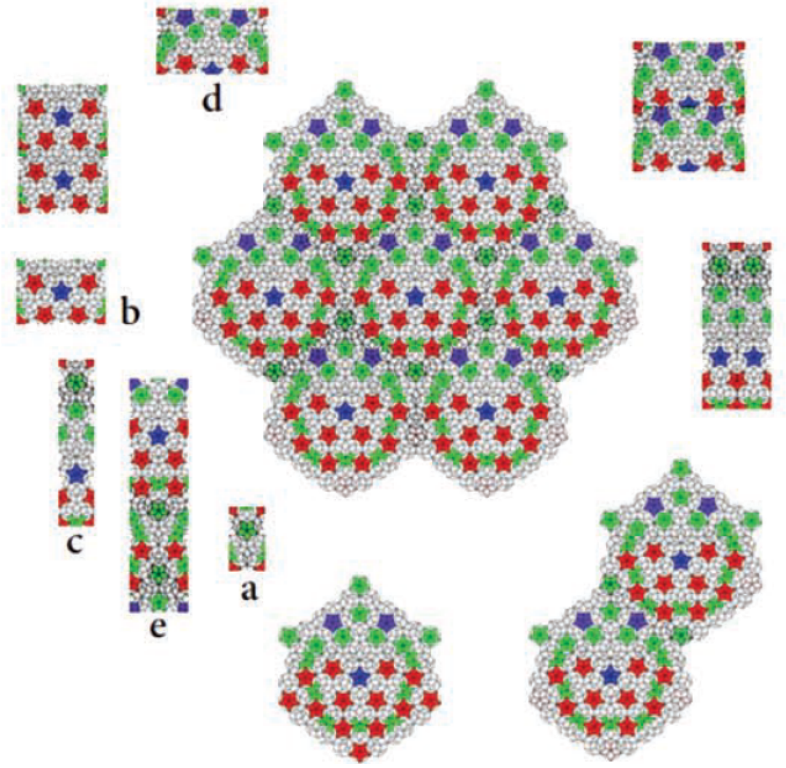

Figure 8: Unit cell can be tessellated to construct a translational periodic Penrose tiling

Apparently, The tiles treated with conjugate (complementary) coupling described in this article are no longer five-fold symmetrical, but a mirror symmetrical.

In fact, in figure 8 a considerable number of (elemental) unit cells can be extracted from their parent tile, and they can all be tessellated individually to construct a new tile (crystal) that exhibits a different structure from its parent tile (crystal). This basic unit cell may be very useful in unlocking the mysteries of quasicrystals.

Similar to rolling up different types of carbon nanotubes, four different Penrose tubes can be fabricated by rolling up a translating Penrose tiles along the x-direction, or along the y-direction, or along some tilt angle, depend on different unit cell. A three-dimensional Penrose unit ball may be constructed by performing rolling on a unit cell. These were all covered in another publication [Kung, 5].

Summary

In this article, two distinct Pentagon-shaped Penrose tiles are constructed using the original Penrose tiling pattern. It is found that these two Pentagonal-shaped Penrose tiles can be coupled successfully without any defect and at different depths. An elementary unit cell can be extracted from repeated coupling results, and this unit cell can be recombined (tessellated, mosaiced) to form translational periodic Penrose tiles.

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